CMSC 317: The Computational Image

Assignment 7: A Simple 3D Viewer

Create a Processing sketch that visualizes a set of 3D points. You should implement both orthographic and perspective projections. You might use the face dataset [1] demonstrated in class. Your sketch should allow the user to pan, rotate and scale the scene interactively using the mouse or keyboard. The camera matrix which projects the three-dimensional world coordinates into two-dimensional image coordinates can be decomposed into two parts: the extrinsics matrix (a 3D Euclidean transformation $[\mathbf{R}|\mathbf{T}]$) and the intrinsics matrix ($\mathbf{K}$). $\mathbf{K}$ can further be decomposed into the camera parameters and the projection model.

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \mathbf{K}[\mathbf{R}|\mathbf{T}] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} s_x & 0 & c_x \\ 0 & s_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \mathbf{\Pi} \qquad \mathbf{\Pi}_{\text{persp}} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \mathbf{\Pi}_{\text{ortho}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation Matrices in 3D

When rotating a point about the origin in the two dimensional plane we used the following rotation or euclidean tranformation matrices:

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [\mathbf{R}|\mathbf{T}] = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

In three dimensions, when we rotate a point there are three possible planes (xy, yz, xz) and thus three angles of possible rotation. Rotating around the z-axis, or in the xy-plane is exactly the transformation we used in the 2D case. The other two rotation matrices correspond to rotating about the x and y axes.

$$\mathbf{R}_z = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R_y} = \begin{bmatrix} \cos\psi & 0 & \sin\psi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\psi & 0 & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R_x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To rotate a point about the origin with angles ϕ,θ,ψ you multiply the matrices in the following order for the product $\mathbf{R}$:

$$\mathbf{R} = \mathbf{R_z}\mathbf{R_y}\mathbf{R_x}$$

[1] http://tosca.cs.technion.ac.il/data/face.zip